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Computational Fluid Dynamics Validation Issues in Transition Modeling

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Laminar-turbulent transition is highly initial- and operating-condition dependent. Finding careful, archival experiments for comparison is the main validation issue for computational fluid dynamics (CFD) modeling. The CFD formulations validated to date demonstrate that if the environment and operating conditions can be modeled and input correctly, the computations (nonlinear parabolized stability equations and direct numerical simulations) agree quantitatively with the experiments. Future challenges for validation include successful CFD simulations of other available complete databases, CFD leadership in the identification and modeling of the effects of freestream disturbances, CFD leadership in the determination of relevant validation experiments for supersonic and hypersonic flows, careful validation experiments and CFD solutions for complex three-dimensional geometries, and simulations and validations for the high Reynolds numbers of flight.

I. Introduction

A. Motivation

A STRONG international interest in problems of stability and transition in wall-bounded shear layers exists in connection with the design of gas-turbine-engine blades and vanes, low-Reynolds-number vehicles, submarines and torpedoes, subsonic and supersonic civil transports and fighters, and hypersonic and re-entry vehicles. Evidence of this is the number of recent meetings, courses, and workshops devoted to the topic of stability and transition. Various recent AGARD special courses provide important sources of information on the aerodynamic application of transition.

Understanding transition is necessary for the accurate prediction of aerodynamic force (lift and drag) and heating requirements. In low-Reynolds-number applications, a significant portion of the flow is transitional, and it is therefore impossible to predict surface phenomena such as drag and heat transfer unless the flow in the transition region is understood. Being off by a factor of 2 in the location of the transition point can make a significant difference in these flows. Moreover, delaying transition by the various techniques of laminar flow control (LFC) generally results in lower drag and therefore higher fuel efficiency.[1–6] Much of the early work is summarized by Pfenninger[7,8] and Reshotko.[9,10] It has been estimated that if laminar flow could be maintained on the wings of a large transport aircraft, a fuel savings of up to 25% would be obtained.[8,11,12]

Of interest to the turbulence community is the fact that boundary-layer flows are open systems, strongly influenced by and dependent on freestream and wall conditions. Breakdown has been well documented to vary considerably when the operating conditions change.[13–15] The transition process then provides the vital upstream conditions from which the downstream turbulent flowfield evolves, and it is reasonable to imagine that different transition patterns give rise to different turbulence characteristics downstream.

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B. Scenario of Transition

Transition in wall-bounded shear layers occurs because of an incipient instability of the basic flowfield, which depends intimately on subtle, and sometimes obscure, details of the flowfield. In other words, the wall-bounded shear layer is an open system. Disturbances in the freestream, such as sound or vorticity, enter the boundary layer as steady and/or unsteady fluctuations of the basic state. This part of the process, called receptivity,[16,17] provides the vital initial conditions of amplitude, frequency, and phase for the breakdown of laminar flow. The recent progress in this area is summarized in Refs. 18 and 19.

Initially these disturbances may be too small to measure, and they are observed only after the onset of an instability. Except in the case of Klebanoff and Görtler modes, the initial growth of these disturbances is described by linear stability theory (LST). Arnal[20] and Reed et al.[21] review this subject.

For two-dimensional boundary layers, Tollmien–Schlichting growth is weak, occurs over a viscous length scale, and can be modulated by pressure gradients, mass flow, temperature gradients, etc. As the amplitude grows, three-dimensional and nonlinear interactions occur in the form of secondary instabilities.[22–24] At this point, disturbance growth is very rapid (now over a convective length scale) and breakdown to turbulence occurs quickly.

On the other hand, for three-dimensional boundary layers, e.g., swept wings, and Görtler problems (concave surfaces), nonlinear distortions of the basic flow may occur early on due to the action of the primary instability. These flows are characterized by an extensive distance of nonlinear evolution with eventual saturation of the fundamental disturbance. leading to the strong amplification of very high frequency inflectional instabilities and breakdown.[12,25–33] All the computational fluid dynamics (CFD) issues and challenges for Görtler problems are reviewed in Ref. 30 and therefore will not be reviewed here.

At times, the initial disturbance can be so strong that the growth of linear disturbances is bypassed[16,34–36] and turbulent spots or secondary instabilities occur and the flow quickly becomes turbulent. This phenomenon has been documented in cases of roughness and high freestream turbulence.[6,17,37–39] In this case, transition-prediction schemes based on linear theory fail.

C. Objectives

The focus of this paper is CFD-validation issues in transition modeling; this paper serves as a complement to the paper by Herbert[40] on transition prediction. Here we distinguish between *verification* and *validation*. Per the designations of Roache,[41] we consider *verification* to mean "confirming the accuracy and correctness of the code," i.e., is the grid resolved, are there any programming errors in the codes, etc. Validation requires verification of the code in addition to confirmation of the adequacy of the equations used to model the physical problem. Strictly speaking, a code can be validated only by comparison with quality experimental data.

The three most widely used transition tools are LST, parabolized stability equations (PSEs), and direct numerical simulation (DNS). Section II describes the formulations of the various tools. Section III reviews the state of validation, the range of applicability of the various tools, and the validation challenges. A summary is given in Sec. IV.

II. Formulations

In this section we present very brief descriptions of the three widely used numerical approaches for modern transition problems. Due to space limitations, the formulation of governing equations associated with each tool is not presented here, but is detailed in Ref. 27. Our main focus is the stability and transition of boundary-layer-type flows.

For transition analysis, equations governing the disturbance are typically solved separately from the basic state. The basic-state formulations are not presented or discussed here; however, the validity of these formulations must also be considered because the transition process is known to be sensitive to subtle changes in the basic state. The numerical accuracy of the basic state must be very high because the stability and transition results will be very sensitive to small departures of the mean flow from its exact shape. The stability of the flow can depend on small variations of the boundary conditions for the basic state, such as freestream velocity or wall temperature. Therefore, basic-state boundary conditions must also be very accurate. See the discussion and examples in Refs. 20 and 42.

A. Linear Stability Theory

Linear stability theory has been the most widely used approximate method for stability analysis.[20,21] In this approach the total flow is separated into a steady basic state and an unsteady disturbance. The basic state, by definition, satisfies the equations governing the total flow and represents the flow that exists in the absence of any environmental disturbances. This allows the basic-state and disturbance equations to be solved separately. For LST the basic state is approximated as locally parallel so that the wall-normal velocity component is set to zero and the remaining flow quantities are functions of the wall-normal direction only. The disturbances are assumed to be small enough to allow the nonlinear terms to be neglected. With these approximations and homogeneous boundary conditions, the disturbance equations feature coefficients that depend on the wall-normal coordinate only, and thus the separation of variables into normal modes is possible and an eigenvalue problem results.

The state-of-the-art transition-prediction design tool involves linear stability theory coupled with an e^N transition-prediction scheme and is applied at all speeds.[2,20,21,43–48] The quantity N is obtained by integrating the linear growth rate from the first neutral-stability point to a location somewhere downstream on the body, but e^N represents nothing more than an amplitude ratio. The basic design tool is the correlation of N with transition Reynolds number for a variety of observations. The correlation will produce a number for N (for example, 9) that is now used to predict transition Reynolds number for cases in which experimental data are not available. However, because initial disturbance amplitude is not accounted for, this method is suspect, and the study of receptivity promises a significant advance in practical transition-prediction schemes.

B. Parabolized Stability Equations

In recent years the PSEs have become a popular approach to stability analysis owing to their inclusion of nonparallel and nonlinear effects.[49,50] For linear PSEs (LPSEs), a single monochromatic wave is considered as the disturbance, which is decomposed into a rapidly varying wave function and a slowly varying shape function. The explicit streamwise second-derivative term is neglected based on physical arguments. There still remains the matter of the ambiguity in streamwise dependence; applying a normalization condition ensures that any rapid changes in the streamwise direction will be absorbed by the wave function so that the shape function will vary slowly in this direction. The resulting system of disturbance equations is parabolic and thus requires boundary and initial conditions. To complete the problem formulation, initial values of the disturbance flow quantities must be specified at some streamwise location for the start of the analysis. If the analysis begins in a region where the initial disturbance amplitudes are small, the LST can be used to obtain these initial conditions. The nonlinear PSEs (NPSEs) are derived in a fashion similar to LPSEs with the exception that each disturbance quantity is transformed spectrally in the spanwise and temporal directions.

C. Direct Numerical Simulation

DNS plays an increasingly important role in the investigation of transition. This trend will continue as considerable progress is made in the development of new, extremely powerful computers and numerical algorithms. In such simulations, the full Navier–Stokes equations are solved directly by employing numerical methods, such as finite difference, finite element, or spectral methods. An excellent review is that of Kleiser and Zang,[51] and the AGARD lecture on spatial simulations by Reed[52] serves as a complementary companion.

Transition is a spatially evolving process, and the spatial DNS approach is widely applicable because it avoids many of the restrictions that usually have to be imposed in other methods and is the closest to modeling experiments. For example, no restrictions with respect to the form or amplitude of the disturbances have to be imposed because no linearizations or special assumptions concerning the disturbances have to be made. Furthermore, this approach allows the realistic treatment of space-amplifying and -evolving disturbances as observed in laboratory experiments. The temporal DNS, by contrast, uses periodic boundary conditions in the streamwise direction (identical inflow and outflow conditions) and follows the time evolution of a disturbance as it convects through the flow; upstream influence is limited by this assumption. Moreover, in temporal simulations, the basic state is assumed to be strictly parallel, that is, invariant with respect to the streamwise coordinate. All of these restrictions noted are especially suspect when considering complex geometries, three-dimensional boundary layers, receptivity, and control. Fasel[53] makes similar points.

The basic idea of the spatial simulation is to disturb an established basic flow by forced, time-dependent perturbations. Then the reaction of this flow, that is, the temporal and spatial development of the perturbations, is determined by the numerical solution of the complete Navier–Stokes equations. This method usually requires a large amount of computer resources (CPU and memory) for solution. Moreover, just as in the PSE approach, the use of DNS is hampered by our current lack of knowledge of the connection between the freestream and the boundary-layer response. A physically appropriate upstream or inflow condition must be specified.

D. Code Verification

As mentioned in the Introduction, verification implies confirming the accuracy and correctness of the code. There are mainly three sources of error in the abstraction of continuous PDEs to a set of discrete algebraic equations: 1) discretization errors, 2) programming errors (bugs), and 3) computer roundoff errors. The objective of code verification is then to completely eliminate programming errors and to confirm that the accuracy of the discretization used in solving the continuous problem lies within some acceptable tolerance. Aside from specifying single or double precision, the code developer has little control over the computer roundoff errors, but these are usually several orders of magnitude smaller than the discretization error and far less than the desired accuracy of the solution.

In this section we address programming and discretization errors. Many methods are discussed in the literature for code verification using grid refinement, comparison with simplified analytical cases, etc. For recent discussions see Refs. 41 and 54. Specific suggestions for testing a CFD code for the study of transition include

1) performing grid-refinement studies, 2) solving test problems for which the solution is known, 3) systematically varying the "far-field" boundary locations, 4) comparing linear growth rates, neutral points, and eigenfunctions with linear stability theory, 5) running the unsteady code with time-independent boundary conditions to ensure that the calculations remain steady, and 6) running geometrically asymmetric codes with symmetric conditions.

In addition to the usual code verification techniques, there is a general method that we use to verify the discretizations and locate programming errors by comparison with manufactured analytical solutions.[55] This method is general in that it can be applied to any system of equations. Although it is an extremely powerful tool, this method has received little acclaim in the literature. For clarity, the technique is demonstrated on the Poisson equation:

$$Lu \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x, y) \tag{1}$$

To solve this problem we discretize the operator L using some appropriate approximation (finite differences, spectral, etc.). In general, the exact solution is not available. Therefore, for verification purposes, we force the solution to Eq. (1) to be some combination of analytical functions with nontrivial derivatives. For example, we might consider the system $g \equiv Lv = 5e^{3y}\sin(2x)$, which has an analytical solution $v = e^{3y}\sin(2x)$. The exact solution can then be compared with the computed solution. Of course, manufactured solutions should be chosen with topological qualities similar to those anticipated for the solution to the real problem, e.g., gradients close to the wall. Proper choice for the manufactured solutions also allows the discretization of the boundary conditions to be verified. For large systems of equations a symbol manipulator is recommended for computing g. If a bug occurs, zeroing the coefficients of some terms in Eq. (1) can help to isolate the bug.

III. Status and Challenges of Validation

As mentioned in the Introduction, validation is defined as encompassing verification of the code as well as confirming that the equations used to model the physical situation are appropriate. In this section we discuss to what extent each of the three methods—LST, PSEs and DNS—has been validated. The basis of validation is assumed to be a successful comparison with the few careful, archival experiments available in the literature.

A. Transition in Two-Dimensional Basic States

1. Secondary Instabilities and Transition Mechanisms in Two-Dimensional Boundary Layers

Available experimental databases. There are different possible scenarios for the transition process.

1) Fundamental mode breakdown: The occurrence of three-dimensional phenomena in an otherwise two-dimensional flow is a necessary prerequisite for transition.[56] Such phenomena were observed in detail by Klebanoff et al.[57] and were attributed to a spanwise differential amplification of Tollmien–Schlichting (T–S) waves through corrugations of the mean boundary layer accompanied by streamwise vorticity. The process leads rapidly to spanwise alternating peaks and valleys, i.e., regions of enhanced and reduced wave amplitude, and an associated system of streamwise vortices. The peak-valley structure evolves at a rate much faster than the (viscous) amplification rates of T–S waves. This represents the path to transition under conditions similar to those of Klebanoff et al.[57] and is called a K-type breakdown. The lambda vortices are ordered in that peaks follow peaks and valleys follow valleys. Since the pioneering work of Nishioka et al.,[58,59] it is accepted that the basic transition phenomena observed in plane channel flow are the same as those observed in boundary layers. Therefore, little distinction will be made here as to whether work was done in a channel or a boundary layer. From the theoretical and computational viewpoint, the plane channel is particularly convenient because the Reynolds number is constant, the mean flow is strictly parallel, certain symmetry conditions apply, and one is able to apply temporal theory. Thus progress was first made with the channel-flow problem.

2) Subharmonic mode breakdown: Different types of three-dimensional transition phenomena observed[13,60–64] are characterized by staggered patterns of peaks and valleys and by their occurrence at very low amplitudes of the fundamental T–S wave. This pattern also evolves rapidly into transition. Hot-wire measurements in these experiments show that the subharmonic of the fundamental wave (a necessary feature of the staggered pattern) is excited in the boundary layer and produces either the resonant wave interaction predicted by Craik[65] (called the C-type) or the secondary instability of Herbert[66] (called the H-type). Spectral broadening to turbulence with self-excited subharmonics has been observed in acoustics, convection, and free shear layers and was not identified in boundary layers until the preliminary results of Kachanov et al.[60] This paper reinitiated the interest in subharmonics and prompted the simultaneous verification of C-type resonance.[61,62] Subharmonics have also been confirmed for channel flows[64] and by direct integration of the Navier–Stokes equations.[67,68]

Corke and Mangano[69] and Corke[15] introduced controlled three-dimensional subharmonics along with the two-dimensional fundamental. Only then could detailed measurements be made of the disturbance flowfield. By using segmented heating elements, it is possible to phase shift a signal to each element and to create an oblique wave at any angle or frequency. Then the two-dimensional fundamental and the three-dimensional subharmonic are a simple electronic superposition.[70] As a result, the Corke subharmonic experiments contain the most complete and reliable set of data on subharmonic breakdown. Both chordwise and spanwise variations of the fundamental and subharmonic are given. Corke[15] gives several possible interactions. Another example of the richness of this work is the disturbance streamlines that are reconstructed from numerous profiles. These measurements are taken at different chord locations but at the same point in the oscillation cycle. One sees an increase in intensity toward the wall as the measurements move downstream. These are data that will positively challenge the NPSE and DNS work. The space in this report is not sufficient to cover all of the different types of behavior that are part of the subharmonic breakdown process. The reader is encouraged to go to the original references.

Linear stability theory. Comparisons between the data of the mean-flow and disturbance-flow rms measurements from a single-frequency experiment and between the data from the Blasius solution and a solution from linear stability theory are available in the literature.[71–73] The agreement between theory and experiment is very good and suggests that the two-dimensional basic-state and disturbance problem is well understood, even in situations where control is applied at levels comparable to those being considered for LFC.

As a transition-prediction tool, the e^N method is certainly the most popular technique used today. It works within some error limits only if comparisons are made among experiments with identical disturbance environments. Because no account can be made of the initial disturbance amplitude, this method will always be susceptible to large errors and should be used with extreme care. When bypasses occur, this method does not work at all. However, there is no other practical method currently available for industrial applications.

Another problem with LST is that difficulties in the computation of the N factor increase with the complexity of the basic mean flow. There is practically no problem for two-dimensional, incompressible flows because oblique waves do not need to be taken into account, and the only choice is between temporal and spatial theory. When the Mach number increases, the oblique waves become the most unstable ones, so that the wave orientation constitutes a new degree of freedom. The problem is considerably more complex in three-dimensional flows with the possible coexistence of streamwise and crossflow disturbances.

Parabolized stability equations. To date the PSEs have been applied to a variety of two- and three-dimensional flow situations and are generally regarded as appropriate for connectively unstable flows.[26–29,49,50,74–81]

Bertolotti et al.[80] validated the PSE approach for T–S two-dimensional disturbances in a Blasius boundary layer by comparison with the DNS results of Bayliss et al.[82] For this nonlinear case they compared not only the growth rates but also the mode shapes of the harmonics and found excellent agreement. Three-dimensional NPSE stability results were compared with the experimental results of Kachanov and Levchenko,[62] but only qualitative agreement was achieved. The differences are attributed to virtual leading-edge and slight pressure gradient effects in the experiment. Comparison of the same NPSE results with DNS results of Fasel et al.[83] and of Crouch[84] show much better agreement. The experimental results of Cornelius[85] for K-type transition are compared with NPSE results, and satisfactory (qualitative) agreement is found.

Direct numerical simulation. Computations by Fasel et al.[83] and Fasel[53] under the conditions of Klebanoff et al.[57] showed poor agreement of the spatial growth at peak and valley stations, until a small streamwise pressure gradient was added to the computations. This pressure gradient was present in the experiments of Klebanoff et al. Rist and Kachanov[86] made very detailed comparisons of the results of hot-wire measurements and DNS for K-type breakdown and showed very good agreement of the spatial disturbance development, the disturbance spectra, the instantaneous velocity traces, and the local frequency/spanwise-wave number spectra. By detailed quantitative comparisons of experimental measurements and results of DNS, they have generated a validated database that may be used for the validation of different theories. They are beginning the evaluation of the late-stage data.

Joslin and Streett[87] simulated the subharmonic experiments of Kachanov and Levchenko[62] and found good qualitative agreement. There were some model discrepancies observed between the two, though, which seemed to be resolved when the computations included a small adverse pressure gradient and a small effective frequency variation in the disturbance.

These results and the reviews given by Herbert,[49] Reed,[52] and Kleiser and Zang[51] show that the details of the initial stages of transition for the case of initially weak disturbances (of the T–S type) in a two-dimensional boundary layer are rather well understood. The details of the later stages of transition are still unresolved, but unless one wished to carry the calculations to turbulence, they may not be as important as the initial stages. In this regard, another next step in the simulation validation will be to predict the growing body of detailed data being developed by Nishioka et al.[59,88] and Nishioka and Asai[89,90] on the latter stages of the breakdown process.

A surprise that results from the analytical model of Herbert[66,91] and the Navier–Stokes computations of Singer et al.[14,92] is that, under amplitude conditions of the experimentally observed K-type breakdown, the subharmonic H-type is still calculated to be the dominant breakdown mechanism instead of the fundamental mode. This is in contrast to Klebanoff's experiment, confirmed by Nishioka et al.,[58,59] Kachanov et al.[60] Saric and Thomas,[13] Saric et al.,[63] and Kozlov and Ramazanov,[64] where only the breakdown of the fundamental into higher harmonics was observed. Only Kozlov and Ramazanov observed the H-type in their channel experiments and only when they artificially introduced the subharmonic.

This apparent contradiction was resolved by Singer et al.[14] Under the initial conditions of a forced two-dimensional T–S wave and random noise, the subharmonic mode is present as predicted by theory but not seen experimentally. However, when streamwise vorticity (as is present in the flow from the turbulence screens upstream of the nozzle) is also included, the subharmonic mode is overshadowed by the fundamental mode (as in the experiments!). Here is a case in which the computations have explained discrepancies between theory and experiments. In the presence of streamwise vorticity, the fundamental mode is preferred over the subharmonic; this agrees with experimental observations but not with theory (which does not account for this presence). Without streamwise vorticity, the subharmonic modes dominate as predicted by theory and confirmed by computational simulations. In the presence of streamwise vorticity characteristic of wind-tunnel experiments, the K-type instability dominates and the numerical simulations predict the experimental results. Thus, each experiment is naturally contaminated with low-level streamwise vorticity that provides the background three dimensionality that leads to the secondary instabilities.

2. *Three-Dimensional Point-Source Disturbances*

Another important class of experiments deals with the point source within the boundary layer as a generator of three-dimensional disturbances. The classic experiment in this area is that of Gaster and Grant.[93] In this work, wave packets are created by impulse disturbances that are introduced at the wall. The initial impulse excites many different modes that are selectively amplified and can undergo interference within the boundary layer. Thus one sees a three-dimensional packet of waves that grows and spreads in the flow direction. Gaster and Grant[93] show a series of hot-wire traces at different downstream distances along the centerline, and the shape of the disturbed region is shown in contour plots. Wave number–frequency data are also given with enough detail for theoretical modeling. See Ref. 94 for recent efforts (the DNS comparisons are reviewed in Ref. 52).

The work of Gaster and Grant[93] provided the foundation for looking at stability and transition phenomena that were not initiated by two-dimensional waves and, as such, it is one of the important breakdown mechanisms that must be considered. One cannot rank the importance of different initial conditions that lead to transition because experience has shown that almost anything can happen.

Breuer and Haritonidis[95] and Breuer and Landahl[96] did linear experiments along with linear theory and temporal DNS for large disturbances in the case of an impulsive point disturbance. The linear results look classical, although no direct comparison between theory and experiment is given. The nonlinear calculations are useful in that they illustrate the presence of strongly inflected velocity profiles that could give rise to secondary instabilities.

3. *Large-Amplitude-Disturbance Input*

Comparing their work with the experimental results of Suder et al.,[38] Sohn and Reshotko,[39] and Karlson and Johansson,[97] Rai and Moin[98] simulated transition and turbulence on a straight-line flat plate. These computations required 400 h of CPU time on a Cray YMP, and it was suggested by the authors themselves that a grid still finer by as much as factors of 2 and 1.5 in the streamwise and spanwise directions, respectively, would be required to achieve an accurate simulation.

For a coarse mesh, Yang and Voke[99] predicted transition in qualitative agreement with high-freestream-turbulence experiments.[100]

These last two studies seem to suggest that underresolved DNS solvers may possibly still capture the gross or mean features of transition. But this is not yet clear, and the present authors suggest *caution even if your goal is just a qualitative or ballpark prediction* of transition location. If the details of the transition process are important, then this approach will more than likely give misleading results. Another observation with these high-freestream-turbulence computations is that there is definitely a need for accurate descriptions and cataloging of the freestream environment because transition has been well documented to be highly sensitive to this.

4. *Receptivity in Two-Dimensional Basic States*

At the present time no mathematical model exists that can predict the transition Reynolds number on a flat plate. One obvious reason for this lack is the variety of influences such as freestream turbulence, surface roughness, sound, etc., which are difficult to quantify experimentally yet may trigger transition through a forced response of the flow as a nonlinear oscillator. A second reason, of course, is the poor understanding of the free response of this nonlinear oscillator, i.e., of the fundamental mechanisms that lead initially small disturbances to transition.

Often, freestream disturbances inherent to experiments, such as freestream vorticity, cannot be measured, and yet we discovered in Sec. III.A.1 that when they become entrained, their amplitude and spectral characteristics inside the laminar viscous layer strongly influence which type of transition occurs. It was demonstrated that the computations can predict very different breakdown behavior unless the inflow and boundary conditions match closely those of the experiment. At first glance, the situation seems hopeless. How do you quantify an effect you can't even measure? How do you ever hope to establish a database for CFD validation when you can't provide precise initial and boundary conditions?

Here is an area in which spatial DNS excels and can lead the way for experimental investigations. By systematically applying different conditions along the edges of the computational domain and identifying those quantities in the boundary layer that are indicative of the forcing, the computationalist will be providing valuable input and guidance to the experimentalists as to what quantities can and should be measured. Amplified quantities in the boundary layer are often larger than their predecessors in the freestream and therefore measurable, thus providing a validation database.

For natural receptivity in two-dimensional boundary layers, the complete receptivity question as to how freestream disturbances enter the boundary layer requires consideration of a combination of all the effects, including, for example, roughness, geometry, associated pressure gradients (both favorable and adverse), vibrations, sound, and freestream turbulence, and it is here that computations by spatial DNS can excel. A variety of different geometric conditions and freestream disturbances can be implemented with this technique and the response of the boundary layer quantified and cataloged.

1) Leading-edge effects: With the spatial DNS, finite curvature can be included in the leading-edge region—a feature that was left out of some early unsuccessful receptivity computations. Use of an infinitely thin plate (a straight line) to study leading-edge effects computationally, although popular, is strongly discouraged. The attachment-line or stagnation region is a critical source of receptivity as large streamwise gradients occur there, and an infinitely thin plate features infinite vorticity there (per the simple Blasius solution). No computational simulation can resolve infinite vorticity. By stipulating the plate to have finite curvature at the leading edge, the singularity there is removed and a new length scale is introduced.

Experimentally, the most popular receptivity model has been the flat plat with an elliptic leading edge. Thus it is reasonable that computation models consider the same geometry for validation. However, the curvature at the juncture between the ellipse and the flat plate is discontinuous and provides a source of receptivity.[101] Lin et al.[102] introduced a new leading-edge geometry, a modified superellipse (MSE). This shape has the advantage of continuous curvature (zero) at the juncture with the flat plate. The MSE has the further advantage of having a nose radius and geometry (hence a pressure distribution) close to that of an ordinary ellipse. Computationally, use of a C-grid rather than an H-grid is recommended to avoid singularities in the metric terms in the sensitive nose region. Again, it is important to include and to resolve the attachment-line region accurately.

For low-speed flows, freestream-sound wavelength is typically one or two orders of magnitude larger than instability wavelengths in the boundary layer. Careful leading-edge receptivity experiments suggested for CFD validation are conducted by Saric et al.[103] in the Arizona State University (ASU) unsteady wind tunnel on a zero-pressure-gradient flat plate. The objective of the work is to examine how T–S waves are generated within the boundary layer under the influence of external acoustic forcing in the absence of any other receptivity mechanisms besides the leading edge.

The characteristic length scale for freestream spanwise vorticity is the convective wavelength, which is approximately three times that of the amplified T–S wave at that frequency. Receptivity to freestream turbulence is a crucial problem that has resisted a straightforward solution on the experimental side. We know that an increase in freestream turbulence increases the amplitude of T–S waves and lowers the transition Reynolds number. One should hope that the mechanism is linear for at least part of the range. However, this is not the case, and this is only one part of the difficulty.

The effects of vorticity and/or freestream turbulence on a laminar boundary layer have been studied by a number of investigators. In particular, over the past several years Kendall[104–109] has developed a careful and well-thought-out experimental setup where he can systematically control freestream turbulence and examine the initiation of T–S waves in the boundary layer.

In related work, Parekh et al.,[110] in a very clever experiment, attempt to have a transverse vortical flow in the freestream interact with the leading edge to create T–S waves. They use an oscillating array of ten ribbons upstream of the leading edge to generate a periodically varying vortical flow. This disturbance flow is swept over a plate. Besides the leading edge, the plate also has a forward-facing step as a receptivity element. In spite of using all of the caution and care required of such an experiment, they are unable to see T–S waves from either the leading edge or the step. The theoretical work of Kerschen[111] and the DNS work of Buter and Reed[112] indicate that a vortical flow should indeed create T–S waves. Needless to say, the study of freestream vorticity is an unusually difficult experiment, and there is a need for companion DNS assistance to sort out the different observations and to establish a validation database.

Whereas the freestream-sound receptivity continues to yield results that can be compared with theory and computations, the freestream-turbulence problem has put up stiff resistance in spite of the skills of the experimenters.

2) Roughness: Spalart[113] studied the effects of small two-dimensional roughness under the flow conditions of the tape experiments of Saric et al.[114] and the theory of Crouch.[115] For all conditions considered, the computation, theory, and experiments agree. The theory and calculations of Goldstein,[116] Kerschen,[117] Kerschen et al.,[118] Bodonyi,[119] Crouch,[115,120] Choudhari and Streett,[121] and Nayfeh and Ashour[122] seem to indicate that the two-dimensional roughness receptivity problem is very well understood.

Spencer et al.[123] worked on experiments with three-dimensional roughness elements with the idea of validating the work of Kerschen et al.[118] and Tadjfar and Bodonyi.[124] Because of the difficulty of measuring close to the roughness element, the near-field results are not reliable, and only moderate agreement with the theory was reached in the far field.

Wlezien and Wiegel[125] considered distributed two-dimensional roughness as receptivity in an effort to provide a database for the theoretical work of Choudhari[126] and Crouch.[127] Tadjfar and Reda[128] look at a wavy surface that is part of the same problem.

B. Transition in Three-Dimensional Basic States

1. *Available Experimental Databases*

This section addresses the crossflow instability that causes the breakdown to turbulence in three-dimensional boundary layers that are characteristic of swept-wing flows. The review of Saric[12] provides an extensive list of references for the recent experiments, including the DLR experiments in Germany on a swept flat plate, a Russian swept-flat-plate experiment, the Centre d'Etudes et de Recherches de Toulouse (CERT)/ONERA experiments on swept wings, the Institute of Fluid Science (IFS) work in Sendai on cones and spheres, and the ASU swept-wing experiments. The archival databases appropriate for CFD validation include the following. The DLR efforts in Germany are reported in Refs. 129–133. The CERT/ONERA efforts in France are reported in Refs. 134–138. The Russian efforts are summarized in Ref. 139. The ASU work is summarized in Refs. 25, 32, and 140–144. The IFS work in Sendai is described in Refs. 145–149. These papers established the existence of both traveling and stationary crossflow vortices, saturation of the stationary crossflow vortex, the nonlinear secondary instability leading to transition, and the sensitivity to freestream disturbances and surface roughness. Here are some great challenges to the computationalist.

One of the key missing ingredients in all three-dimensional boundary-layer experiments is the understanding of receptivity. Receptivity has many different paths through which to introduce a disturbance into the boundary layer and this road map is more complicated because of the amplified stationary vortices. In fact, many aspects of transition in three-dimensional boundary layers are orthogonal to two-dimensional boundary layers and so such a road map is either not unique or too complicated. Aside from the usual mechanisms, such as the interaction of freestream turbulence and acoustical disturbances[150] with model vibrations, leading-edge curvature, attachment-line contamination,[151] discontinuities in surface curvature, etc., the presence of roughness that may enhance a stationary streamwise vortex is very important. In contrast to two-dimensional boundary layers where small two-dimensional roughness is important and three-dimensional roughness is less important unless it is large, the three-dimensional boundary layer appears to be very sensitive to micron-sized three-dimensional roughness. In this case, two-dimensional roughness is only important at its edges.

2. *Linear Stability Theory*

Linear stability theory is not so successful for three-dimensional boundary layers. Arnal[20] reviews and provides references for the determination of the spanwise wave number and the most suitable path for integration in the e^N method. When applied to available flight and wind-tunnel experiments, there is large scatter in the values of the N factor at the onset of transition among the different methods. There are three reasons for this behavior. 1) Transition location is more difficult to determine accurately in three-dimensional flow than in two-dimensional flow. 2) Crossflow disturbances are very sensitive to micron-sized roughness elements, which have no effect on streamwise disturbances.[143] 3) Disturbance development can be dominated by nonlinearities during a large part of the transition process, and the use of linear theory up to breakdown is inappropriate and can overestimate wave amplitude.[12,25–33] (Schrauf[152] also discusses different N-factor integration strategies.)

Whereas linear stability theory predicts that the traveling crossflow waves are more amplified than the stationary crossflow waves, many experiments observe stationary waves. The question of whether one observes stationary or traveling crossflow waves is cast inside the receptivity problem. Müller and Bippes,[33] Bippes and Müller,[132] and Bippes[130] describe a series of comparative experiments in a low-turbulence tunnel and a high-turbulence tunnel. Their results show that traveling crossflow waves are observed in the high-turbulence tunnel rich in unsteady freestream disturbances, and the dominant structure in a low-turbulence tunnel is a stationary crossflow vortex. Because the flight environment is more benign than the wind tunnel, one expects the low-turbulence results to be more important.

One of the important results to come out of the DLR group is the set of data that show early saturation of the disturbance amplitude and the failure of linear theory to predict the growth of the instability. They also report distorted mean profiles, similar to those of Michel et al.,[153] Dagenhart et al.,[142] and Kohama et al.,[32] due to the presence of the stationary corotating vortices. A similarity between the DLR and ASU experiments is the high N factors and the high amplitude of the mean-flow distortion (10–20%). It is not surprising that linear theory fails.

For low-amplitude crossflow waves, Radeztsky et al.[144] find that linear stability theory correctly predicts the expected wavelengths and mode shapes for stationary crossflow. For this case, Haynes and Reed[29] find that linear theory including curvature correctly predicts the growth rates. As discussed in the following section, this is not the case for higher-amplitude crossflow, and the results of Reibert et al.[25] and Haynes and Reed[28] demonstrate conclusively that a nonlinear calculation is required to obtain complete agreement.

Note that the strength of linear theories is in their use for design by comparing growth rates and N factors from one configuration to another or by doing parametric studies. A configuration with a smaller N factor (using the same form of the theory) is likely to remain laminar longer. If the theory at least qualitatively contains the appropriate relationships, it can be a practical and efficient tool in the evaluation of new airfoil shapes for wings.

3. Parabolized Stability Equations

The NPSE approach has recently been validated for three-dimensional flows subjected to crossflow disturbances in Refs. 26–29. Here a detailed comparison of NPSE results with the experimental measurements of Reibert et al.[25] show remarkably good agreement. The configuration is an NLF(2)-0415 45-deg-swept airfoil at −4-deg angle of attack. A spanwise array of roughness elements is used near the airfoil leading edge to introduce 12-mm spanwise-periodic crossflow disturbances into the boundary layer. The initial conditions for the NPSE calculation were obtained by solving the local LST equations at a 5% chord location for the fundamental [mode (0, 1)] and adjusting its rms amplitude such that the total disturbance amplitude matched that of the experiment at 10% chord.

Figure 1 shows the comparison of experimental N-factor curves with LPSE, NPSE, and LST. The NPSE results include curvature effects. It is clear that the linear theories fail to accurately describe the transitional flow for this situation. After a region of linear growth, the disturbance modes achieve large amplitudes and interact nonlinearly, saturating at about 30% chord location. There is a large region of nonlinear interaction from 30–50% chord before transition occurs.

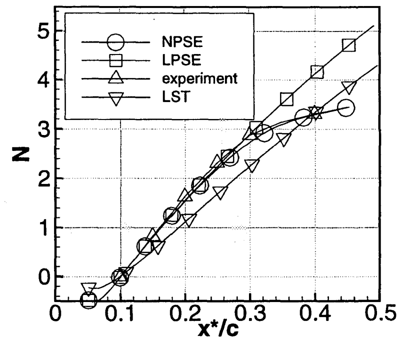

Fig. 1 ***N*** **factors for NLF(2)-0415 45-deg-swept airfoil: $R_c = 2.4 \times 10^6$ and $\lambda_z = 12$ mm.**

These results validate the NPSE approach for three-dimensional crossflow-dominated transition cases where the disturbance inputs are known or controlled. Unfortunately, the disturbance inputs for flight conditions are not known. However, Schrauf et al.[74] point out that transition information can be obtained by comparing results using initial conditions from standard environments to perform trade-off analyses.

4. Direct Numerical Simulation

The DNS has also been validated for three-dimensional boundary layers. Müller et al.[154] performed two simulations under conditions close to those of the DLR swept-wing experiment.[33] In one simulation, a steady vortex was generated, which developed in a fashion very similar to the vortex-dominated experiment. For a simulation with a vortex–wave interaction, the steady vortices saturate at a significantly lower level than for the experiment.

The NPSE and DNS have both shown very encouraging results in validating against the available experimental databases, but more work is still needed to simulate physical initial conditions.

C. Supersonic/Hypersonic Flows

In contrast to incompressible flows, there is no guidance from experiments regarding the nonlinear stages of transition in supersonic and hypersonic flows. There is no comparable Klebanoff experiment. Initial CFD efforts have indicated that the amount of resources required can far exceed an incompressible calculation; gradients of disturbance quantities are generally steeper, and compressibility is known to reduce disturbance amplitudes, thus delaying the normal appearance of breakdown in a given computational box. DNS results are only available for the simple geometry of the flat plate. Supersonic and hypersonic flow are instances in which CFD (NPSE and DNS) must lead and guide what experiments should be performed and measurements taken for validation.

For this reason we concentrate on the basic fundamental differences between subsonic and supersonic streamwise instabilities that have already been documented from computational investigations. The paper by Mack[43] is the most complete description of compressible stability available anywhere. The linear stability analysis of supersonic boundary layers uncovers three major differences between it and the subsonic analysis: the presence of a generalized inflection point, multiple acoustic modes (Mack modes), and the dominance of three-dimensional viscous disturbances.

Thumm et al.,[155] Bestek et al.,[156,157] and Leib and Lee[158] studied spatially growing three-dimensional waves in a growing two-dimensional flat-plate boundary layer; in the computations the disturbances were introduced via periodic wall blowing/suction. They pointed out that a secondary instability calculation based on a finite two-dimensional amplitude may not be relevant for supersonic flow, and they investigated other possible routes to turbulence at low supersonic Mach numbers. To this end they simulated a Mach 1.6 base flow subjected to a pair of three-dimensional waves of amplitude $\mathcal{O}(1\%)$ and discovered a new breakdown mechanism, termed oblique-wave breakdown. The disturbances quickly became nonlinear and, through direct nonlinear interactions, a strong longitudinal vortex system was observed. The resulting structures, which differed from the Λ-shaped vortices usually reported for fundamental or subharmonic breakdown, were described as honeycomb-like.

Hypersonic flows are even more complicated for some of the following reasons. 1) At hypersonic speeds, the gas often cannot be modeled as perfect because the molecular species begin to dissociate due to aerodynamic heating. In fact, sometimes there are not enough intermolecular collisions to support local chemical equilibrium, and a nonequilibrium-chemistry model must be used. 2) The bow shock is very close to the edge of the boundary layer and must be included in studies of transition. Stuckert and Reed[159] analyzed the stability of a shock layer in chemical nonequilibrium and compared results with the flow assuming 1) local chemical equilibrium and 2) a perfect gas. It is clear that the equilibrium and nonequilibrium solutions can differ significantly. A complete quantitative description of the effects of the finite shock-layer thickness on transition modeling awaits either a PSE solution or a DNS analysis.

D. Recommendations for Experimental Databases

To this end, as a message to those performing experiments, it is critical for an experimentalist to completely document the flowfield as a companion data set to transition measurements. This includes physical properties, background disturbances, initial amplitudes, and spatial variations.[160]

Saric[160] also suggests that regardless of whether the experimental objectives are transition control; three-dimensional, secondary instabilities; nonlinear breakdown; or receptivity, the linear problem must be correct. That is, if one can show the comparison with linear theory for the particular flowfield, then the experimental basic state is probably as advertised. A measurement of transition Reynolds number is also advised.

IV. Summary

Transition is highly initial-condition and operating-condition dependent. Validation requires comparison with careful archival experiments, but few such experiments have been performed. The good news is that the CFD formulations validated to date demonstrate that if the environment and operating conditions can be modeled and input correctly, the computations (NPSE and DNS) agree quantitatively with the experiments. What is especially significant and exciting is that the NPSE, which have significantly less resource overhead associated with them compared with DNS, have been shown to accurately model a variety of relevant flows.

Challenges for validation include the following: 1) successful CFD simulations of available databases, for example, the Corke and Mangano combination–resonance experiments and the Nishioka et al. plane-channel experiments; the Kendall, Parekh, Wlezien, and Saric receptivity experiments; the Arnal, Bippes, Kachanov, and Saric three-dimensional boundary-layer experiments; and the Reshotko nonuniform basic-state experiments; these include NPSE and DNS; 2) CFD leadership in the identification and modeling of the effects of freestream disturbances; 3) CFD leadership in the determination of relevant validation experiments for supersonic and hypersonic flows using NPSE and DNS; 4) careful validation experiments and CFD solutions for complex three-dimensional geometries including the complete transition to turbulent flow, bypasses including three-dimensional distributed roughness, and better understanding and specification of the freestream environment; and 5) simulations and validations for the high Reynolds numbers of flight.

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